Full-Vector Beam Propagation Methods for 3D Optical Waveguides

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Introduction
An optical waveguide is a dielectric structure which confines and guides light waves to propagate along a pre-determined path. An optical fiber is a well-known optical waveguide. The beam propagation method (BPM) is the most widely used numerical tool for simulating the propagation of light in 3D optical waveguides. We develop a wide-angle full-vector BPM based on alternating direction implicit (ADI) pre-conditioner. The exponential of a square root operator (the propagator) is first approximated by a rational function of the transverse differential operator $X$. The linear equations involving the operator $X$ are solved by a Krylov subspace method with an ADI pre-conditioner. To show the validity and usefulness of our methods, waves propagating in a Y-branch rib waveguide is numerically simulated.

Basic Formulations
1.) Consider the one-way equation for the transverse components of the magnetic field,

$$\frac{\partial u}{\partial z} = ik_0 n_* \sqrt{1 + X} u$$

where $u = [H_x, -H_y]^T$ is a vector related to the $x$ and $y$ components of the magnetic field, $X$ is a $2 \times 2$ transverse differential operator, $k_0$ is the free space wave number and $n_*$ is a reference refractive index.

2.) Equation (1) is discretized in the propagation direction $z$. For a propagation step from $z_j$ to $z_{j+1} = z_j + h$,

$$P u_{j+1} = Pu_j + h X u_j$$

where $P$ is the propagator, $h$ is the step size, $s = k_0 n_* h$, $X$ is evaluated at the midpoint $z_j + h/2$, $u_j$ approximates $u$ at $z_j$, etc.

3.) The $[p-1/p]$ Padé approximants of $P$ can be used and this requires solving the linear equation

$$(1 + bX)w = f$$

where $b$ is a given constant, $f$ is a given vector of two functions and $w$ is a vector of two unknown functions.

4.) Reformulate equation (3) as in the ADI iterative scheme for elliptic problems, then solve the new equation by a Krylov subspace method called BICGSTAB. Also, the optimal value of a parameter is calculated form eigenvalue estimates of the related operators.

Numerical Example
- The Y-branch is excited by the fundamental mode field for $\lambda = 1.55 \mu m$.
- Propagates from $z = 0$ to $40 \mu m$ with $\Delta z = 0.1 \mu m$.
- Computational domain: $|x| < 4 \mu m$ and $|y| < 2 \mu m$.
- Discretized with grid sizes $\Delta x = \Delta y = 0.05 \mu m$.
- Perfectly Matched Layer with thickness = 0.3 $\mu m$.
- $[3/4]$ Padé approximants of $P$ is used with $n_* = 3.3885$
- Magnitude of $H_y$ at $z = 0$, 24 and 40 $\mu m$ are shown below: